A New Method of Solving the Bernoulli Quadrisection Problem

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Problems that involve the minimisation of certain geometrical quantities have attracted the attention of mathematicians since at least the seventeenth century. The famous Fermat-Torricelli Problem dates from that era. This problem requires us to find the point in the interior of a triangle such that the sum of the distances from that point to the vertices of the triangle is a minimum. Another famous problem of this type is Fagnano's Problem which asks us to find the triangle of least perimeter that is enclosed in a given triangle. The Bernoulli Quadrisection Problem is not as well-known as these and has never, to the best of our knowledge, been viewed as a problem involving minimisation. It involves the division of the area of a given triangle into four equal areas, one a triangle and the other three being quadrilaterals, using two perpendicular straight lines. We show that the Bernoulli Quadrisection Problem can be recast as a minimisation problem and we use a modern, very efficient minimisation algorithm (Particle Swarm Optimisation) to solve the problem.

Although our project was inspired by problems that have a long history in Mathematics, we also discuss a new method of solving a very modern problem. This problem was proposed and solved by Professor Jens Vygen in 2005. It has applications in the design of electronic circuits using Very Large Scale Integration (VLSI). We present a very different method of solution based on Particle Swarm Optimisation (PSO).

In our work on the Vygen problem we used a form of PSO that involves multiple swarms. The use of multiple swarms is certainly not new, but neither is it particularly common. This experience encouraged us to think about the possibility of designing new and improved PSO algorithms. The literature on PSO suggests that slow convergence can sometimes be a problem, and that there is a risk of convergence to a local rather than a global minimum. In the standard PSO algorithm, the values of the so-called 'cognitive' and 'social' coefficients are constant. We have devised a modified PSO algorithm in which these coefficients depend on the value of the objective function for each particle. A hybrid algorithm, combining our new algorithm with Spall's Simultaneous Perturbation Stochastic Approximation Optimisation algorithm, has also been programmed and tested. Very significant improvements in efficiency, compared to the standard PSO algorithm, have been achieved with both of these new algorithms.