Abstract

Bidiagonal Decompositions of (Singular) Vandermonde-type Matrices

The standard formulas for the decompositions of the Vandermonde-type matrices are undefined in the case of a singular matrix, which happens when two nodes are equal. In this paper, we presented new formulas for computing the bidiagonal decompositions (factorizations) of the Vandermonde, (q-,h-) Bernstein-Vandermonde, h-Bernstein-Vandermonde, Rational Bernstein-Vandermonde, Cauchy-Vandermonde with one multiple pole, and Lupas matrices.

We obtained new explicit formulas for the nontrivial elements of the factorizations. The method we used consists of factoring the diagonal matrix of the standard decomposition into multiple diagonal matrices and 'inserting' them into the lower bidiagonal matrices, canceling out all the denominators of type 'x-y', as they could possibly equal 0. All the nontrivial entries of the matrices of the factorizations can be put into two matrices: one contains the subdiagonal elements of the lower bidiagonal matrices, the diagonal elements of the diagonal matrix, and the superdiagonal elements of the upper bidiagonal matrices; the other contains the diagonal elements of the lower and upper bidiagonal matrices, as they are no longer unit lower and unit upper bidiagonal, respectively.

The new formulas allow for accurate computations to be made with the matrices, even when they are singular. Matlab programs were created for this purpose - they compute the decompositions at a quadratic computational complexity given just the nodes of the matrices as a parameter (the user does not need to enter the whole matrix), and print them out in the form of the aforementioned form, ready to be used for computations. Using these formulas, eigenvalues can be computed with the correct sign and first 14 decimal digits. The correct number of zero Jordan blocks (Jordan blocks corresponding to eigenvalue 0) can also be computed.

The method for changing the factorizations we presented can also be applied to create new bidiagonal decompositions for other Vandermonde-type matrices, such as the (Rational) Said-Ball and the (Wronskian-) Jacobi matrices.